

Examination cover sheet

(to be completed by the examiner)

Course name: Diff. vgl. en Math. Course code: 2D BAI

Date: 06-04-2016

Start time: 18.00 End time: 21.00

Number of pages: 4

Number of questions: 5

Maximum number of points/distribution of points over questions: 40

Method of determining final grade: total points/4

Answering style: formulation, order, foundation of arguments, multiple choice:

Exam inspection: _____

Other remarks: _____

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify: _____

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will **in any case** be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

**Examination Differentiaalvergelijkingen en Matrices (2DBA1)
on 06 April 2016, 18.00-21.00 hours.**

- Formulate the computations and the results of the exercises in a clear way.
 - It is not allowed to use a laptop, graphical calculator or a chart with formulas.
 - It is not allowed to use a book or other handwritten material.
 - The mobile telephones are not stand by, they are put in a bag. Going to the toilet is only allowed without taking your mobile telephone with you.
 - The order in which questions will be resolved is entirely free.
 - The exam consists of 5 exercises.
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Opmerking: $\exp(x) = e^x$.

1. Given are the complex numbers, with $i^2 = -1$,

$$z_1 = \exp\left(-\frac{1}{3}\pi i\right), z_2 = 2 \exp\left(\frac{2}{3}\pi i\right).$$

- a. Draw the numbers z_1 en z_2 in the complex plane.
b. Let

$$z_3 = (2(z_1)^2 - (\frac{z_2}{2})^2).$$

Calculate the exact values of $\operatorname{Re}(z_3)$ and $\operatorname{Im}(z_3)$.

- c. Let

$$z_4 = (z_1 + z_2)^6.$$

Calculate the exact values of $|z_4|$ and $\arg(z_4)$.

- d. Let

$$z_5 = \left(\frac{z_2}{2z_1}\right)^{2017}.$$

Calculate the exact values of $\operatorname{Re}(z_5)$ and $\operatorname{Im}(z_5)$.

SEE NEXT PAGE

2. Given is the following boundary-value problem:

$$\cos(x) \frac{dy}{dx}(x) = \sin(x) y(x) + 2 + e^x$$

$$\text{with } -\frac{\pi}{2} < x < \frac{\pi}{2} \text{ and } y(0) = \pi.$$

- a) Determine the solution of the homogeneous differential equation.
- b) Determine the solution of the given initial-value problem.

3. The matrix A is defined by:

$$A = \begin{pmatrix} 1 & 0 & 1 & b \\ a & 1 & a & (a + ab) \\ b & 0 & (a + b) & (1 + b^2) \\ b & 0 & b & (a - ab + b^2) \end{pmatrix}. \quad (1)$$

with the parameters a and b in \mathbb{R} .

Consider the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 1 & b \\ a & 1 & a & (a + ab) \\ b & 0 & (a + b) & (1 + b^2) \\ b & 0 & b & (a - ab + b^2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} a \\ 1 + a^2 \\ 4 + a \\ 1 + a + ab \end{pmatrix}.$$

with the parameters a and b in \mathbb{R} .

- a) For which value(s) of a and b has the linear system exactly one solution? Calculate the associated solution.
- b) For which value(s) of a and b has the linear system no solution?
- c) For which value(s) of a and b has the linear system infinitely many solutions? Calculate the associated solutions, if they exist.
- d) Take $a \neq 0$ and $b = 2$.
Calculate the inverse of the matrix A .
- e) Take $a = 0$ and $b = -1$.
Determine $R(A)$, so determine the range of A .
(Determine the image area of A .)

SEE NEXT PAGE

4. Given is the matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- a) Calculate the eigenvectors of A .
- b) Determine the coefficients $\alpha, \beta \in \mathbb{R}$, such that:

$$(A^3) = \alpha (A^2) + \beta A.$$

- c) Give a matrix S such that

$$S^{-1} A S = D,$$

with D a diagonal matrix, with the eigenvalues of A as its diagonal elements.

5. Given is the following inhomogeneous linear system of first order differential equations:

$$B \frac{d}{dt} \underline{y}(t) = A \underline{y}(t) + \underline{f}(t), \quad (2)$$

whereby

$$A = \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}$$

and

$$\underline{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \text{ en } \underline{f}(t) = \begin{pmatrix} -6 \\ 10 \end{pmatrix} e^{(3t)}.$$

- a) Calculate the general solution $\underline{y}_H(t)$ of the homogeneous system of differential equations.
- b) Find a particular solution $\underline{y}_P(t)$ of the given inhomogeneous system of differential equations.

SEE NEXT PAGE

For the exercises, the following number of points can be achieved:

1 a : 2	2 a : 4	3 c : 2	4 b : 2	
1 b : 2	2 b : 4	3 d : 2	4 c : 3	
1 c : 2	3 a : 2	3 e : 2	5 a : 4	
1 d : 2	3 b : 2	4 a : 2	5 b : 3	

The result of this exam can be calculated by dividing the total number of points by 4, and it will be rounded to one digit behind the comma.

In the result of the interim test (2DBA2) also determine the result of the exam (2DBA0) than:

If all conditions are fulfilled, the result will be:

$\{0.3 * (\text{result of the interim test (2DBA2)})$
 $0.7 * (\text{result of this exam (2DBA1 (> 5.0(!))))\}$
 the result will be rounded to a whole number.